### Quantum interference as a resource for quantum speedup [arXiv:1305.2186]

**Main result**
Consider a quantum circuit that ends with a two-outcome (yes/no) measurement. Limiting to a single unitary for simplicity of presentation, the goal is to estimate \(|\psi\rangle^{U1}M|\psi\rangle\). This can be written as a special case of a more general form, 

\[
|\psi\rangle^{U1}M|\psi\rangle = |\psi\rangle^{ABC}\phi(\psi).
\]

which can be computed via a sum over Feynman-like paths in the computational basis:

\[
|\psi\rangle^{ABC}\phi(\psi) = \sum_{ij} |a_i\rangle|b_j\rangle C_{ij}\phi(\psi).
\]

We define a measure of interference via a sum over absolute values:

\[
\|I\| = \sum_{ij} |a_i| |b_j| C_{ij}\phi(\psi).
\]

The interference producing capacity of an operator is the maximum amount of interference it can produce, which ends up being

\[
I_{\text{max}}(A) = \|\phi(a|A)\|,
\]

where \(a|A\) := |a| and ||\| is the maximum singular value. Modulo some technical details, we have the following theorem:

**Theorem:** The value of \(|\psi|A\cdots Z|\psi\rangle\) can be estimated to accuracy \(\pm \epsilon\) on a classical computer in time \(O(e^{-2I_{\text{max}}(A)^2}) \sim O(\epsilon^{-2})\).

The simulation involves sampling paths chosen according to a convex combination of two Markov chains, one going left-to-right \((\psi \rightarrow A \rightarrow B \rightarrow \phi)\) and the other going right-to-left. This was inspired by, and can be seen as an extension of, [4] which dealt with sparse matrices.

**Open Question:** Can we do it in time \(O(e^{-2I})?\)

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**References**


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**Footnote**
This is what the Haar wavelet transform looks like on three qubits: